CONTRIBUTION TO THE MODELING OF THE CYCLICAL AND MONOTONIC BEHAVIOR OF REINFORCED CONCRETE STRUCTURES

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ABSTRACT

The present paper deals with the behavior of reinforced concrete beams in presence of plasticity and cyclic loadings. The model takes into account the nonlinear material behaviors of the constituents steel and concrete. A numerical model based on the finite element method is investigated for the study of the reinforced concrete beams under cyclic loads undergoing large deformation in the plastic range. In the study, nonlinear material behavior laws are introduced in presence of plasticity and cyclic behavior in the concrete in compression and in the reinforcement steel bars under tension stresses. The concrete steel interactions as well as the crack damage are also considered in the model. An incremental iterative method in adopted in the solution of the equilibrium equations.

The model is validated and compared to some benchmark solutions available in literature. The agreement is good in the case of beams under monotonic loads or cyclic loads with high cycles.

Keywords: Reinforced concrete; cyclic behavior; non-linear; flat structures; plasticity.

1. INTRODUCTION

Under the seismic stresses action, the behavior of reinforced concrete structures can undergo a large incursion outside the linear domain. Moreover, seismic actions impose cyclical and dynamic stresses on the structures.

Consequently, the evaluation of the behavior of the reinforced concrete structures under seismic actions requires a good knowledge of their nonlinear behavior, under monotonic and cyclic loads.

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Analysis of reinforced concrete structures requires realistic constitutive models and analytical procedures in order to produce reasonably accurate simulations of behavior. The research in this field is rich and extensive in theory, numerical investigations and test setup.

Nilson [21] proposed a non-linear analysis of reinforced concrete structures using the finite element method. Bathe's team contribution in this field is very important and enabled the assessment of the behavior of these structures with complex material behavior laws, Bathe and al [4, 5]. The use of popular commercial codes (Abaqus, Ansys, Adina) for numerical simulations for reinforced concrete structures is more frequent (Xu [35], Ahmed [1], Genikomsou and Polak [11]). In addition, several theoretical and experimental studies have been conducted. Franklin [10] developed a study taking into account the effects of material non-linearity, and used a beam element with three-degrees of freedom based on the first order theory of displacements. Grelat [12] proposed a program of calculation of reinforced concrete planar frameworks based on the parabolic diagram for concrete under tension. Filippou and Kwak [9] carried out several studies on the non-linear analysis of reinforced concrete elements under monotonic loads by the finite element method.

Salari and Spacone [28] developed a non-linear model for single-dimensional beam element taking into account the phenomenon of steel-concrete adhesion. Due to the complexity in the behavior and modeling of reinforced concrete structures under cyclic loading, most of the previous works have been concentrated on its behavior under monotonic loading case.

In these works, we are interested in the study and modeling of the non-linear material behavior of reinforced concrete beams under monotonic and cyclic static loads in presence of plasticity. The manuscript is presented as follows. In section 2, the basics of the theoretical model are presented. The material behaviors of the constituents follow in section 3. The solution procedure implanted in the program for the solution of the nonlinear behavior system is presented in section 4. A numerical approach is adopted in the study and the present model is incorporated in a homemade code. In order to illustrate the accuracy and practical usefulness of the proposed model, many examples are considered. The convenience of the model is outlined and discussed. Some remarks are dressed in order to improve the present study. The conclusion and discussions close the paper.

2. BASIS OF THE MODEL

The behavior of a beam element is modeled by studying the behavior of a bending moment zone discretized in a finite number of transversal sections subjected to a non uniform bending moment in order to evaluate the stress-strain state and to determine the stiffness matrix and forces in the cross-section. We adopted the multilayer approach. For this aim, the following assumptions are adopted:

a. Planar beams are considered in the study. The loads are applied in the xz plane where x and z are the axial and the vertical axes. In bending, the beam is subject to displacement $u(x)$ in the axial direction, displacement $w(x)$ in the vertical direction and to rotation $\theta(x)$ (Fig. 1a).

b. The beam deforms according to the plane of symmetry xz. Out plane displacements are then not considered.

c. During deformation, the beam is subject to the stress resultants where $N(x)$ is the axial force, $T(x)$ is the shear force and $M_y(x)$ the bending moment

d. Planar and straight sections before deformation remain plane and orthogonal to the neutral fiber after deformation. The deformations of shear forces are neglected (Navier-Bernoulli model).

e. Second order displacements and deformations are neglected (linear relationship between deformations and displacements).
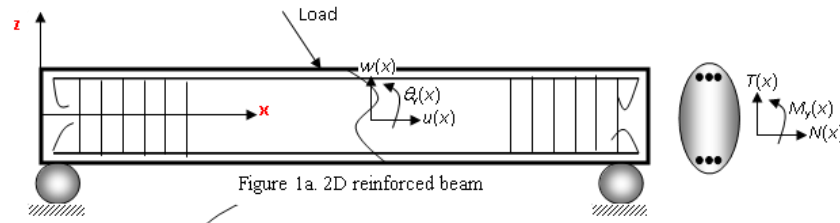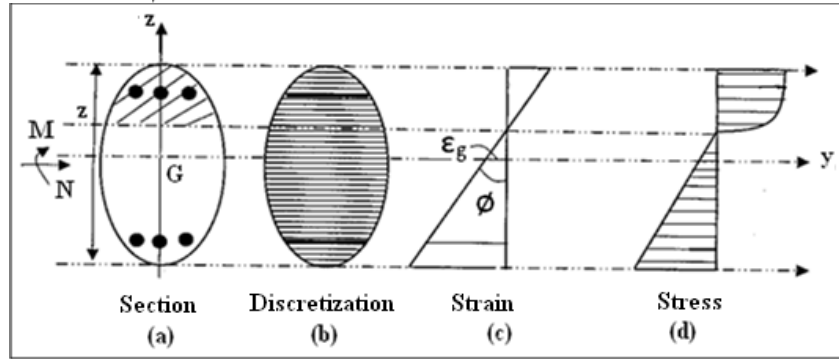

Figure 1a. 2D reinforced beam

Figure 1b. Cross section modelisatio

For the discretization of displacements, a 2D beam finite element with two nodes and 3 degrees of freedom per node is used in the study. The longitudinal displacement $u_0(x)$, at the reference axis, is approximated by Lagrange-type interpolation functions of degree 1 in the polynomial base:

$$u_{0(x)} = \left(1 - \frac{x}{l}\right)u_1 + \left(\frac{x}{l}\right)u_2 \tag{1}$$

For transverse displacement $w_{(x)}$, the classical Hermite interpolation functions of degree 3 are used in the polynomial base:

$$w_{(x)} = \left[1 - 3\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)^3\right]w_1 + \left[3\left(\frac{x}{l}\right)^2 - 2\left(\frac{x}{l}\right)^3\right]w_2$$
$$+ l\left[\frac{x}{l} - 2\left(\frac{x}{l}\right)^2 + \left(\frac{x}{l}\right)^3\right]\theta_1 + l\left[-\left(\frac{x}{l}\right)^2 + \left(\frac{x}{l}\right)^3\right]\theta_2 \tag{2}$$

$u_1, w_1, \theta_1, u_2, w_2, \theta_2$ are the degrees of freedom of the beam (Fig. 2).
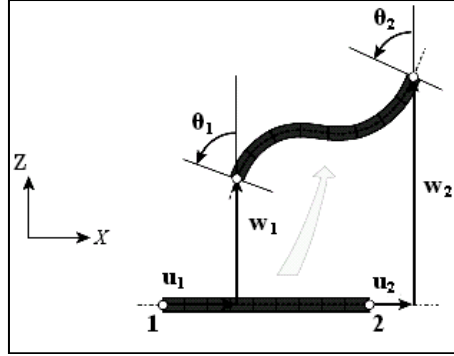

Figure 2. Finite element with two nodes before and after deformation

The axial deformation at the beam element geometric center G is given by:

$$\varepsilon_g = \frac{du_{0(x)}}{dx} \tag{3}$$

The curvature is given by:

$$\emptyset = \frac{-d^2 w_{(x)}}{dx^2} \tag{4}$$

The longitudinal deformation $\varepsilon(z)$ of an horizontal fiber situated at the z ordinate with respect to the axis Gy is defined by means of two parameters (Fig. 1b (c)), namely the deformation ε_g and the curvature $\emptyset$ given by:

$$\varepsilon(z) = \varepsilon_g + z\emptyset \tag{5}$$

The stress resultants in the cross section are defined by the following relationships:

$$N = \int \sigma(z)dA \, , M = \int \sigma(z)zdA \tag{6}$$

By using the secant elasticity modulus E_s of the materials, we obtain a relationship between the cross section forces resultants and the section deformations:

$$\left\{ \begin{matrix} N \\ M \end{matrix} \right\} = \begin{bmatrix} \overline{EA} & \overline{ES} \\ \overline{ES} & \overline{EI} \end{bmatrix} \left\{ \begin{matrix} \varepsilon_g \\ \varnothing \end{matrix} \right\} = [K_s] \left\{ \begin{matrix} \varepsilon_g \\ \varnothing \end{matrix} \right\} \tag{7}$$

Conversely, the relation (7) is written:

$$\left\{ \begin{matrix} \varepsilon_g \\ \varnothing \end{matrix} \right\} = [K_s]^{-1} \left\{ \begin{matrix} N \\ M \end{matrix} \right\} \tag{8}$$

where $[K_s]$ is the secant stiffness matrix of the section.

Under the stress forces N and M, the deformations $\boldsymbol{\varepsilon_g}$ and curvature Ø determined using an iterative non linear method. The components of the stiffness matrix $[K_s]$ are determined by discretising the cross-section into n_b horizontal layers and n_a steel rows.

$$\left\{ \begin{aligned} \overline{EA} &= \int_A E_s dA = \sum_{i=1}^{nb} E_{bi} A_{bi} + \sum_{j=1}^{na} E_{aj} A_{aj} \\ \overline{ES} &= \int_A E_s z dA = \sum_{i=1}^{nb} E_{bi} z_i A_{bi} + \sum_{j=1}^{na} E_{aj} z_j A_{aj} \\ \overline{EI} &= \int_A E_s z^2 dA = \sum_{i=1}^{nb} E_{bi} z_i{}^2 A_{bi} + \sum_{j=1}^{na} E_{aj} z_j{}^2 A_{aj} \end{aligned} \right. \tag{9}$$

3. MATERIAL BEHAVIOR MODELING

Reinforced concrete is a material made of steel and concrete, these two components have very different mechanical responses, in both compression and tension. It is well known that the steel bars exhibit similar behavior either in tension and compression. The concrete behavior is characterized by relatively good strength in compression and very limited strength in tension. It is therefore essential to understand the assumptions made and the laws considered in the definition of the behavior of each of these materials and their combination on the behavior of a reinforced concrete beam. In what follows, the material behavior models of these two constituents are described in presence of cyclic loads and plasticity.

3.1 Concrete behavior
There is a very abundant literature on the laws of cyclic behavior of concrete: Aslani [2], Bahn [3], Benmansour [6], Karsan [15], Palermo [23], Seckin [30], Sima [31], Sinha [32], Yankelevski [34]. Most of these models deal only with cyclical behavior in compression and only few of them takes into account its traction behavior as in Aslani [2], Chen [7], Okamura [22], Palermo [23].

In the present work, the proposed uni-axial model is based on experimental observations. It allows the correct description of the non-linear behavior of the concrete, the decrease of the stiffness, the appearance of the residual deformations and the restitution of stiffness during an alternating loading.

According to Fig. 3 and until a new decrease of the loading, the concrete follows the non-linear compression law of Sargin [29] (Envelope curve, path 1), when the stress sign changes, the concrete discharges along a line of slope E_2 passing through a focal point (f_{bc}, ε_0) as suggested by Park [24]. This decision agrees with tests carried out by Ramtani [26] who showed that in presence of the damage of the compressed concrete, the discharge modulus is different from the initial modulus E_{b0}. In the damaged discharge, the modulus E_2 (with $E_2 \leq E_{b0}$, Path 2, Fig. 3) is then defined by:

$$E_2(i) = \frac{f_{bc} - \sigma_{i-1}}{\frac{f_{bc}}{E_{b0}} - \varepsilon_{i-1}} \tag{10}$$

And the residual deformation is given by equation 11, which is approximately one-fifth of the maximum deformation reached during loading according to tests carried out by Neild and al [19]:

$$\varepsilon_{r(i)} = \frac{f_{bc}}{E_{b0}} - \frac{\frac{f_{bc}}{E_{b0}} - \varepsilon_{i-1}}{1 - \frac{\sigma_{i-1}}{f_{bc}}} \tag{11}$$

when the concrete is loaded in tension, we keep the damaged module E_2 computed previously until the tensile strength f_t is reached. This is in agreement to tests by Morita and Kaku [18] who showed that highly damaged concrete in compression sees its modulus decrease sensibly. Moreover, Chen and Bu [7] observed that the beam stiffness in the unloading states varies with the loading history. Beyond this resistance, the concrete follows the Grelat [12] envelope curve (Path 3).

When the stress changes sign, the crack closes progressively along a line of slope E_1 and follows the Path 4 in Fig. 3. The crack is assumed to be completely closed for a stress less than ($-f_t$) and this is approximately the same stress that Aslani and Jowkarmeimandi [2] took into consideration ($\sigma_f = f_e / 10$), beyond this stress we find the line of slope E_2 of discharge in compression (reloading in compression, Path 5, Fig. 3) and in the case of an initially tensed point we join the nonlinear law of concrete in compression (loading in compression):
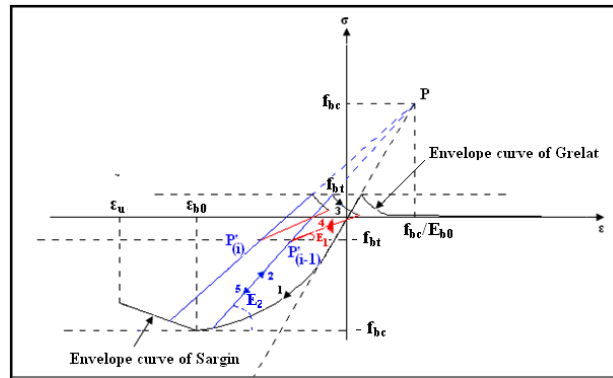

Figure 3. Uni-axial model of concrete

$$E_{1(i)} = \frac{\sigma_{i-1} + f_{bt}}{\varepsilon_{i-1} - \varepsilon_{p'}} \ and \ \varepsilon_{p'} = \frac{f_{bc}}{E_{b0}} - \frac{(f_{bc} + f_{bt})}{E_{2(i-1)}} \tag{12}$$

3.2 Steel material behavior

Steels for the reinforcement increases the stiffness of the concrete in the tensile part and therefore it must be taken into account in the modeling of the reinforced concrete structures.

In the rheological point of view, the adopted models of steel material can be elastic perfectly plastic behavior or an elastic-plastic law with consideration of hardening. In the present study, the elastic perfectly plastic model is adopted:

$$\begin{cases} \sigma_s = E_a \varepsilon & if \ \varepsilon \le f_e/E_a \\ \sigma_s = f_e & if \ f_e/E_a < \varepsilon \le \varepsilon_u \\ \sigma_s = 0 & if \ \varepsilon > \varepsilon_u \end{cases} \tag{13}$$

With the condition:

$$\begin{cases} \sigma_s > f_e & \to \sigma_s = f_e \\ \sigma_s < -f_e \to \sigma_s = -f_e \end{cases} \tag{14}$$

4. SOLUTION PROCEDURE

In the analysis of the non-linear behavior of a structure by the finite element method, we arrive at a system of algebraic equations of the form:

$$\{F\} - [K(U)]\{U\} = \{\psi(U)\} \ne 0 \tag{15}$$

With:

$[K(U)]$: Stiffness matrix of the structure linked displacements to the forces.

$\{U\}$: Vector of the nodal displacement.

$\{F\}$: Vector of the nodal forces applied to the structure.

$\{\Psi(U)\}$: Vector of residual forces.

The stiffness matrix is determined by the assembly of the elementary stiffness matrixes [K]e, which are evaluated by using a numerical integration based on the Gauss method:

$$[K]_e = \int_0^l [B]^t [D][B] dx \tag{16}$$

$$[B] = \begin{bmatrix} \dfrac{-1}{l} & 0 & 0 & \dfrac{+1}{l} & 0 & 0 \\ 0 & (\dfrac{6}{l^2} - \dfrac{12x}{l^3}) & (\dfrac{4}{l} - \dfrac{6x}{l^2}) & 0 & (\dfrac{-6}{l^2} + \dfrac{12x}{l^3}) & (\dfrac{2}{l} - \dfrac{6x}{l^2}) \end{bmatrix} \tag{17}$$

$$D = \begin{bmatrix} \overline{EA} & \overline{ES} \\ \overline{ES} & \overline{EI} \end{bmatrix} \qquad (18)$$

For the solution of the previous equations (15), we have considered an iterative calculation procedure using secant stiffness matrix which consists in finding the solution $\{U\}$ which makes the residue $\{\psi(U)\}$ as close as possible to zero. The non-linear resolution flowchart is shown in Fig. 4.
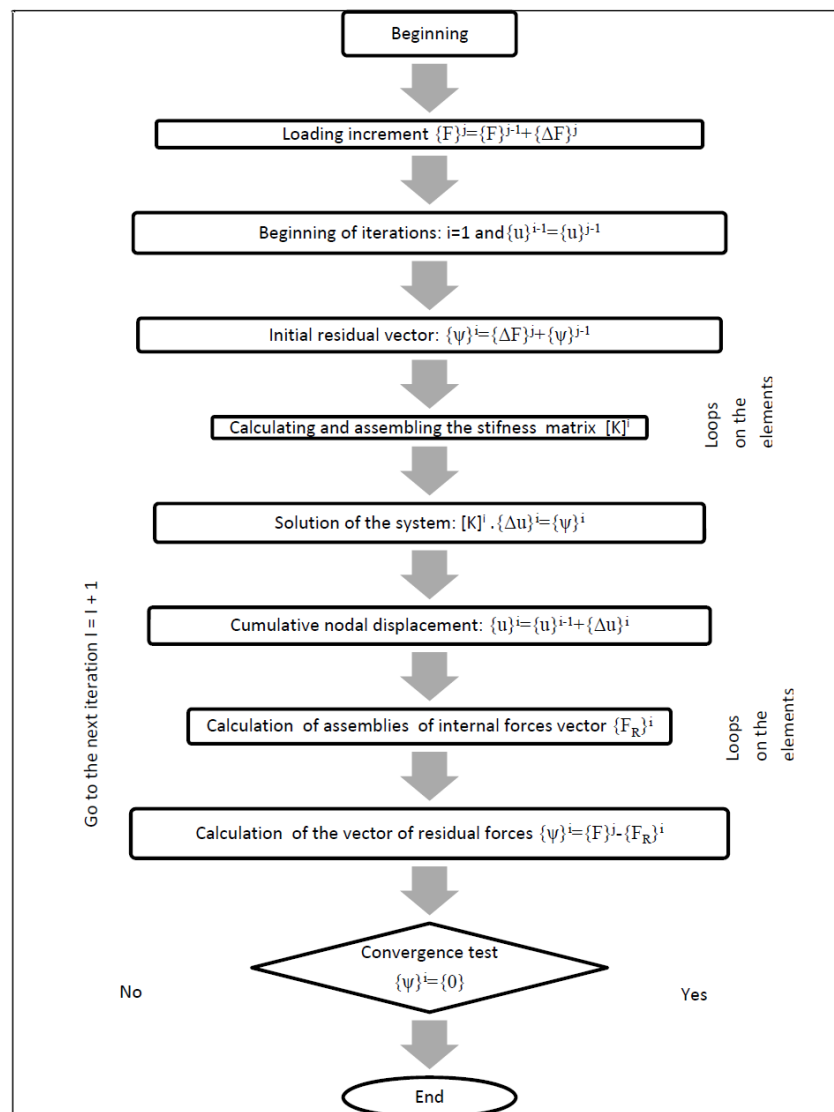

Figure 4. Non-linear calculation flowchart

On the basis of the modeling and calculation methods presented above, we have developed a homemade computer program called "Cycl_beam", written in Fortran 90

language, which allows the numerical simulation of the non-linear static behavior of the reinforced concrete beams under cyclic and monotonic loads. This program has been validated by comparison to some benchmark solutions available in literature. Some examples are studied in the next section.

5. VALIDATION OF THE RESULTS

In the validation procedure, three reinforced concrete beams are considered. For these beams, the results of the numerical simulations are compared to whose experimental data results. The first example investigated the nonlinear behavior under under monotonic loading in presence of plasticity. Two other examples include plasticity and cyclic loading.

5. 1 Example 1*: Structures under monotonic loadings*
It is a highly reinforced beam proposed by Pera [25] and Tuset [33]. The tested beam has slenderness of (2x270 cm). A rectangular cross section (20x50 cm) is adopted. 3 layers of reinforced steel bars (2x3T32) are present in the bottom side. Only one layer (2T8) is present in the top side. The beam is subjected to pure bending moment. The geometrical data are defined in Fig. 5. The characteristics of the materials used are summarized in Table 1. They are taken into account in the numerical simulations.

In Pera [25], an increasing load is applied to the beam until the ultimate limit. The beam is then subjected to monotonic bending moment. In order to make a first validation of the model presented here, a numerical calculation is carried out on the same beam. The numerical response curves are plotted on the experimental curves in Fig. 6 and 7 for comparison. The numerical and experimental load-deflection curves are plotted in Fig. 6. The concrete deformation in terms of the loading is given in Fig. 7.

In the load deflection curve (Fig.6), the experimental behavior is well simulated up to a load of 250 KN. This value seems to be the elastic limit in the experimental study. Beyond this limit, the numerical curve seems slightly more stiffer than the experimental curve. In the numerical simulations, the elastic limit is higher and averages 350 kN. In the load strain curve (Fig. 7), one observes that the numerical and experimental curves seem to have the same tendency and the agreement is good. According to this curve, the elastic load is near 350 kN in both numerical and test curves. This means, that the elastic load is well obtained by the numerical simulations. However, the failure load is well simulated by the 400kN calculation (2% deviation) as well as the corresponding deflection at the failure load. The agreement seems to be acceptable.

The theoretical failure mode is the same as the observed experimentally. In Fig. 7, one observes a crushing of the compressed part at the point of application of the load.
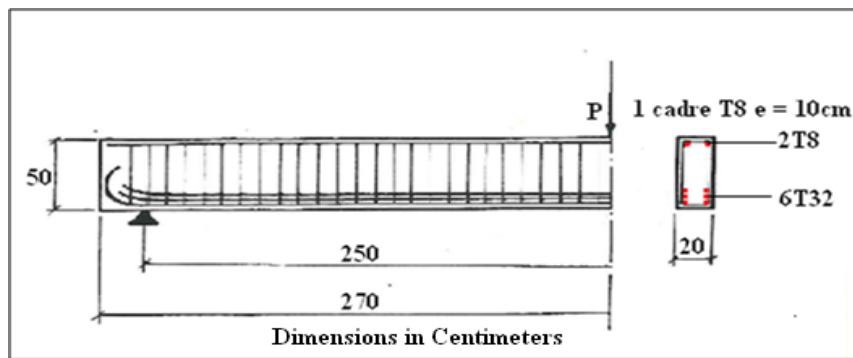
Figure 5. Geometrical data of beam Pera [25]

Table 1: Material data

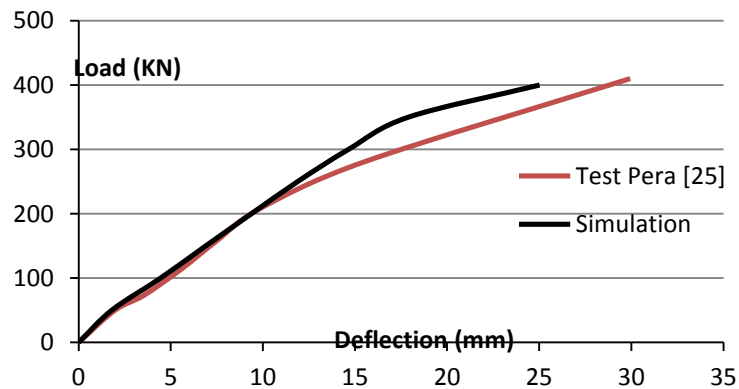| Concrete	Steel
Modulus of initial elasticity: 37600 MPA	Young Modulus: 220000 MPA
Poisson's coefficient: 0.22	Yield stress: 368 MPA
Limit stress in compression: 41 MPA	Failure stress: 488 MPA
Tensile stress limit: 3.9 MPA	Elongation at break : 24%

Figure 6. Numerical and experimental comparison of load-deflection curve

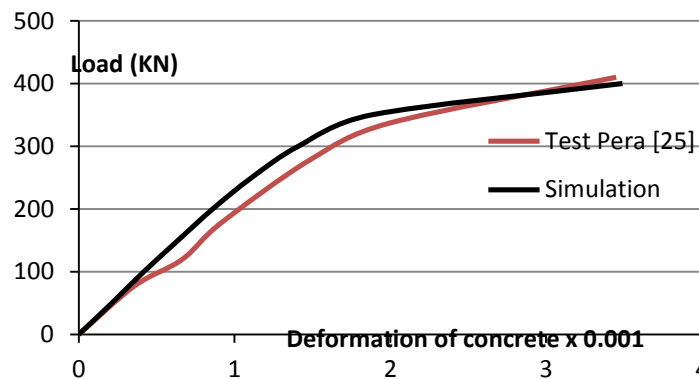Figure 7. Numerical and experimental comparison of Concrete deformation-Load variation

5.2 Structures under cyclic loading
5.2.1 Example 2: Beam behavior under cyclic loads
This is a reinforced concrete beam studied in [17]. A beam with slenderness (4 m) was tested in bending with two equal forces applied at 55 cm from the mid length (Fig. 8). A rectangular cross section (15x25 cm) is adopted. One layer reinforced steel bars (2T25) are present in the bottom and top sides. In the test, the beam was subjected to an alternating bending moments. The geometric data are defined in Fig. 8 and the material data are summarized in Table 2.

Among the numerous results, we show in Figs. 9 and 10 those for the 1st and 2nd cycles. In the first cycle, the beam was tested in compression until a load 16,8 kN and 27 kN in tension. The deflection reaches 6,32 mm in compression and 10,24 mm in tension. In the second cycle, the beam was tested in compression until 52,44 kN in compression and 63.4 kN in tension. The deflections reach 19 mm in compression and 23,33 mm in tension. The results obtained by the present model are very satisfactory and agree well with test results [17]. The overall responses are practically superimposed, all with a slight over-estimation of the loading-unloading slope at the 1st cycle. The agreement of the present numerical model is then good with tests.
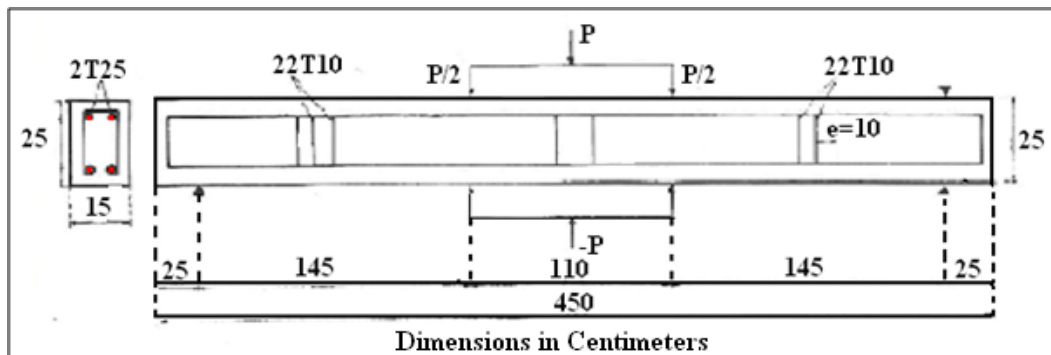

Figure 8. Geometrical data of the beam [17].

Table 2: Material datas

Concrete	Steel
Initial elastic modulus: 35000 MPA	Young Modulus: 200000 MPA
Poisson's coefficient: 0.2	Yield stress: 400 MPA
Limit stress in compression: 28 MPA	
Limit stress in traction: 2.8 MPA	

5.2.2 Example 3: Benchmark beam
A simply supported beam was tested in bending. The beam considered has a rectangular section of 0.2 m thickness and 0.5 m height, and its total length is 5.4 m. A rectangular cross section (20x50 cm) is adopted. Reinforced steel bars (2xT32) are present in the bottom side. (2xT8) are used in the top side. The material data are presented in Table 3.
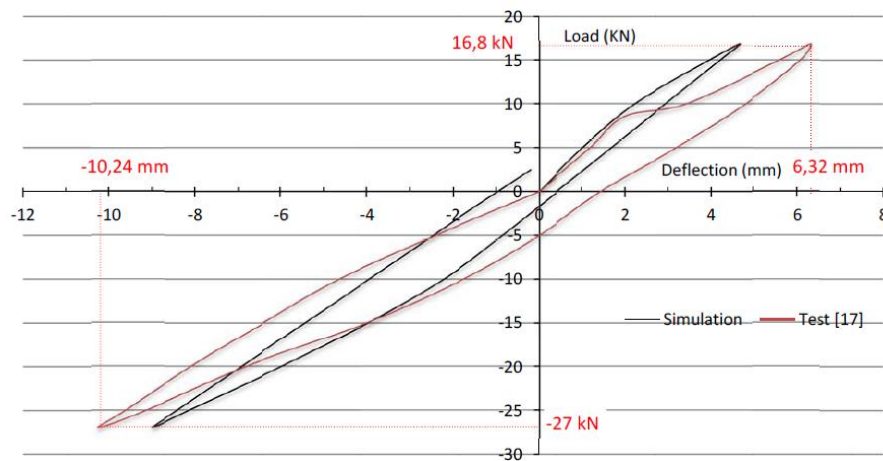
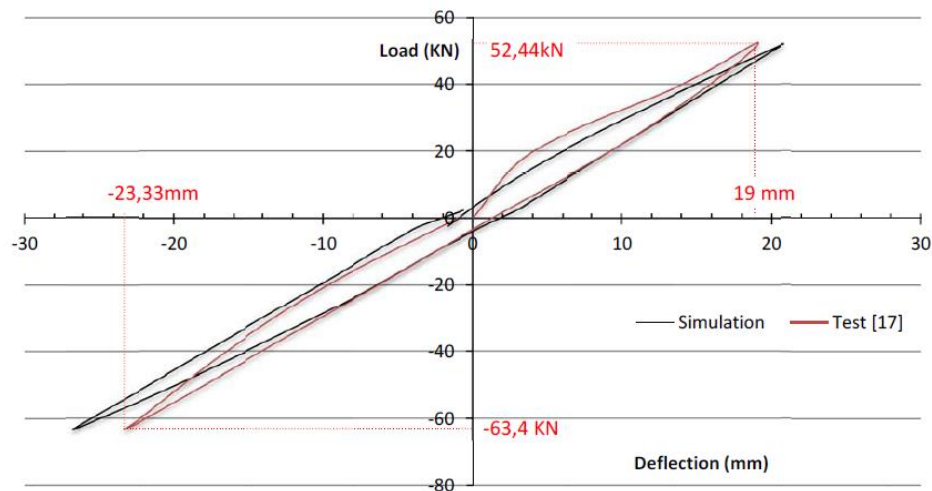Figure 9. Numerical and experimental load-deflection curve in first cycle

Figure 10. Numerical and experimental load-deflection curve in the second cycle.

Table 3: Characteristics of materials

Concrete	Steel
Initial elastic modulus: 37200 MPA	Young modulus : 195302 MPA
Poisson's coefficient: 0.2	Yield stress: 466 MPA
Limit stress in compression: 36.08 MPA	Failure stress: 615 MPA
Limit stress in traction: 3.45 MPA	

The benchmark beam [27] was tested under compressive cyclic loadings only (8 compression cycles, no tension). In order to validate the present model, a numerical study was carried out and the results are presented in Figs, 11 and 12. The global response of the beam under the 8 cycles is presented in Fig. 11. The numerical results of the present model are reported and compared to test results [27]. The displacement response of the beam for cycles 6, 7 and 8 is shown in Figs. 12 (a), 12 (b) and 12(c) respectively.

Once again, the cyclic behavior is well simulated by our program for the 8 loading cycles until the ultimate load which averages 300 KN (Fig. 11). The model used was able to predict successfully the degradation of the beam stiffness from one cycle to another. The beam residual deformation and the deflection values are reproduced well numerically. The agreement with test results is good especially for the last two cycles (Fig. 12 (b) and Fig. 12 (c)).
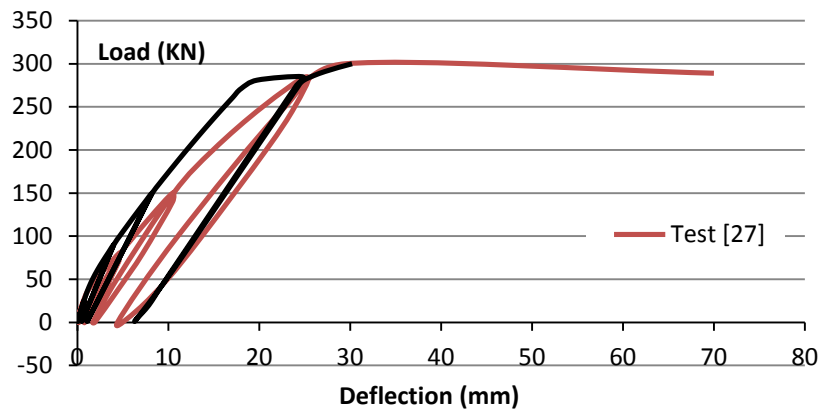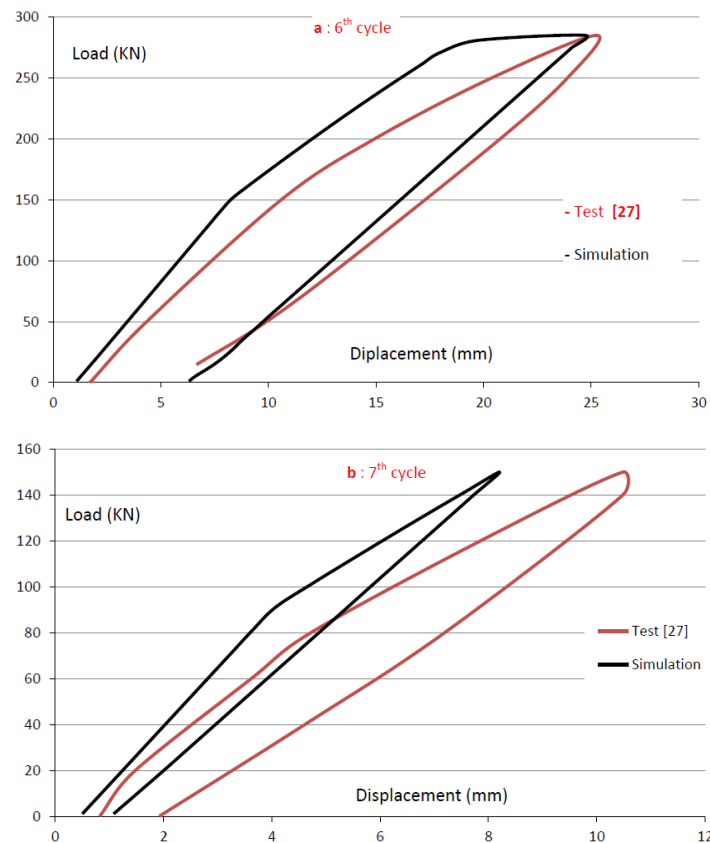

Figure 11. Numerical and experimental load-deflection curve in the first 8 cycles
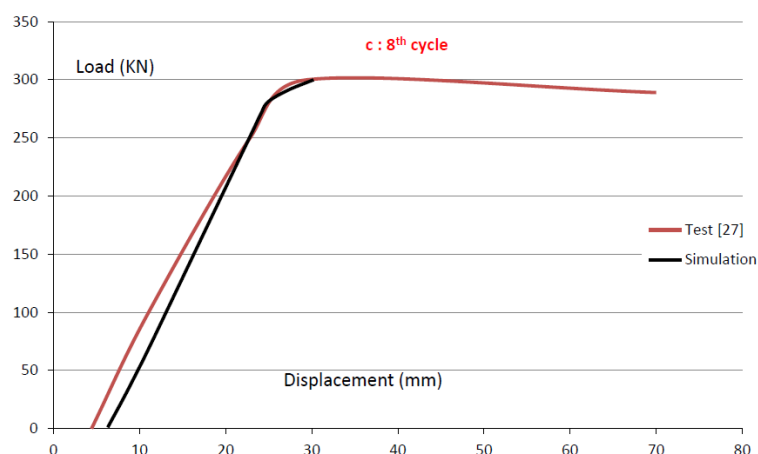
Figure 12 a-c. Numerical and experimental load-deflection curves in the 6th, 7th and 8th cycles

6. CONCLUSION

In this study, we were interested in the nonlinear modeling of the cyclic and monotonic behavior of reinforced concrete beams. The model considers nonlinear material behavior of the constituents; the concrete and the reinforcement steel bars. Effects of plasticity and cyclic loads on beam damage are taken into account in the model.

In this framework, a formulation of a finite element beam as well as the behavior models of steel and concrete materials are presented. A computational tool allowing the numerical simulation of the nonlinear behavior including plasticity and cyclic behavior of plane structures is developed and implanted in a homemade program.

The validation is carried out by comparison of the present model to some benchmark solutions of reinforced concrete beams available in literature. The numerical-test comparison is satisfactory and proves the accuracy of the model developed.

The model of steel is investigated under the assumption of an elastic perfectly plastic behavior. The effects of hardening are ignored in the present model. Moreover, more accurate model is under consideration in order to improve our knowledge of the complex behavior of these structures. One of the possibilities is to include the hardening effects and to adopt a commercial code as comparison. This work is under consideration.

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